Pick someone in your group to join Discord. It's fine if multiple people join, but one is enough.

Now switch to Pensieve:

• **Everyone**: Go to discuss.pensieve.co and log in with your @berkeley.edu email, then enter your group number. (Your group number is the number of your Discord channel.)

Once you're on Pensieve, you don't need to return to this page; Pensieve has all the same content (but more features). If for some reason Penseive doesn't work, return to this page and continue with the discussion.

Post in the #help channel on Discord if you have trouble.

Getting Started

Everyone go around and say your name, just in case someone forgot.

For fun: Think of a big word with at least three syllables, such as "solitary" or "conundrum" or "ominous". Try to use it as many times as you can during today's discussion, but in ways that don't give away that it's your big word. At the end, your group will try to guess each person's big word. Whoever uses their big word the most times (and at least twice) without their group guessing it wins. (You win nothing; it's just a game.)

To get help from a TA, send a message to the discuss-queue channel with the @discuss tag and your discussion group number.

If you have fewer than 4 people in your group, you can merge with another group in the room with you.

The most common suggestion from last discussion was to add some hints, so we have. The second most common suggestion was to encourage more discussion and collaboration. Discuss and collaborate!

Trees

For a tree t: - Its root label can be any value, and label(t) returns it. - Its branches are trees, and branches(t) returns a list of branches. - An identical tree can be constructed with tree(label(t), branches(t)). - You can call functions that take trees as arguments, such as $is_leaf(t)$. - That's how you work with trees. No t == x or t[0] or x in t or list(t), etc. - There's no way to change a tree (that doesn't violate an abstraction barrier).

Here's an example tree t1, for which its branch branches(t1)[1] is t2.

```
t2 = tree(5, [tree(6), tree(7)])
t1 = tree(3, [tree(4), t2])
```

A path is a sequence of trees in which each is the parent of the next.

You don't need to know how tree, label, and branches are implemented in order to use them correctly, but here is the implementation from lecture.



Example Tree

```
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch), 'branches must be trees'
    return [label] + list(branches)
def label(tree):
    return tree[0]
def branches(tree):
    return tree[1:]
def is_leaf(tree):
    return not branches(tree)
def is_tree(tree):
    if type(tree) != list or len(tree) < 1:</pre>
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
```

Q1: Warm Up

What value is bound to result?

result = label(min(branches(max([t1, t2], key=label)), key=label))

How convoluted! (That's a big word.)

Here's a quick refresher on how key functions work with max and min,

max(s, key=f) returns the item x in s for which f(x) is largest.

```
>>> s = [-3, -5, -4, -1, -2]
>>> max(s)
-1
>>> max(s, key=abs)
-5
>>> max([abs(x) for x in s])
5
```

Therefore, max([t1, t2], key=label) returns the tree with the largest label, in this case t2.

In case you're wondering, this expression does not violate an abstraction barrier. [t1, t2] and branches(t) are both lists (not trees), and so it's fine to call min and max on them.

4 Trees

Q2: Has Path

Implement has_path, which takes a tree t and a list p. It returns whether there is a path from the root of t with labels p. For example, t1 has a path from its root with labels [3, 5, 6] but not [3, 4, 6] or [5, 6].

Important: Before trying to implement this function, discuss these questions from lecture about the recursive call of a tree processing function: - What recursive calls will you make? - What type of values do they return? - What do the possible return values mean? - How can you use those return values to complete your implementation?

If you get stuck, you can view our answers to these questions by clicking the hint button below, but *please* don't do that until your whole group agrees.

What recursive calls will you make?

As you usual, you will call has_path on each branch b. You'll make this call after comparing p[0] to label(t), and so the second argument to has_path will be the rest of p: has_path(b, p[1:]).

What type of values do they return?

has_path always returns a bool value: True or False.

What do the possible return values mean?

If has_path(b, p[1:]) returns True, then there is a path through branch b for which p[1:] are the node labels.

How can you use those return values to complete your implementation?

If you have already checked that label(t) is equal to p[0], then a True return value means there is a path through t with labels p using that branch b. A False value means there is no path through that branch, but there might be path through a different branch.

```
def has_path(t, p):
   """Return whether tree t has a path from the root with labels p.
   >>> t2 = tree(5, [tree(6), tree(7)])
   >>> t1 = tree(3, [tree(4), t2])
   >>> has_path(t1, [5, 6])
                                    # This path is not from the root of t1
   False
   >>> has_path(t2, [5, 6])
                                    # This path is from the root of t2
   True
   >>> has_path(t1, [3, 5])
                                    # This path does not go to a leaf, but that's ok
   True
   >>> has path(t1, [3, 5, 6])
                                    # This path goes to a leaf
   True
   >>> has_path(t1, [3, 4, 5, 6]) # There is no path with these labels
   False
    .....
   if p == ____: # when len(p) is 1
        return True
   elif label(t) != ____:
        return False
   else:
        "*** YOUR CODE HERE ***"
```

If your group needs some guidance, you can click on the hints below, but please talk with your group first before reading the hints.

The first base case should check whether p is a list of length one with the label of t as its only element. The second base case should check if the first element of p matches the label of t.

When entering the recursive case, your code should already have checked that p[0] is equal to label(t), and so all that's left to check is that p[1:] contains the labels in a path through one of the branches. One way is with this template:

for ____: if ____: return True return False

Discussion Time! Can the **else** case of **has_path** be written in just one line? Why or why not? You can ignore how fast the function will run. When your group has an answer, send a message to the discuss-queue channel with the @discuss tag, your discussion group number, and the message "Maybe?" and a member of the course staff will join your voice channel to hear your answer and give feedback.

Q3: Find Path

Implement find_path, which takes a tree t with unique labels and a value x. It returns a list containing the labels of the nodes along a path from the root of t to a node labeled x.

If x is not a label in t, return None. Assume that the labels of t are unique.

First talk through how to make and use the recursive call. (Try it yourselves; don't just click the hint button. That's how you learn.)

What recursive calls will you make?

find_path(b, x) on each branch b.

What type of values do they return?

Each recursive call will either return None or a non-empty list of node labels.

What do the possible return values mean?

If find_path(b, x) returns None, then x does not appear in b. If find_path(b, x) returns a list, then it contains the node labels for a path through b that ends with the node labeled x.

How can you use those return values to complete your implementation?

If a list is returned, then it contains all of the labels in the path except label(t), which must be placed at the front.

```
def find_path(t, x):
    .....
    >>> t2 = tree(5, [tree(6), tree(7)])
    >>> t1 = tree(3, [tree(4), t2])
    >>> find_path(t1, 5)
    [3, 5]
    >>> find_path(t1, 4)
    [3, 4]
    >>> find_path(t1, 6)
    [3, 5, 6]
    >>> find_path(t2, 6)
    [5, 6]
    >>> print(find_path(t1, 2))
    None
    .....
    if ____:
        return ____
    ----:
        path = ____
        if path:
            return ____
    return None
```

Please don't view the hints until you've discussed with your group and can't make progress.

If x is the label of t, then return a list with one element that contains the label of t.

Assign path to the result of a recursive call to find_path(b, x) so that you can both check whether it's None and extend it if it's a list.

For a list path and a value v, the expression [v] + path creates a longer list that starts with v and then has the elements of path.

Description Time! When your group has completed this question, it's time to describe why this function does not have a base case that uses **is_leaf**. Come up with an explanation as a group, pick someone to present your answer, and then send a message to the discuss-queue channel with the @discuss tag, your discussion group number, and the message "Found it!" and a member of the course staff will join your voice channel to hear your description and give feedback.

Document the Occasion

For each person, the rest of the group should try to guess their *big word* (from the Getting Started section). The group only gets one guess. After they guess, reveal your *big word* and how many times you used it during discussion.

Please all fill out the attendance form (one submission per person per week).